Exhibit B

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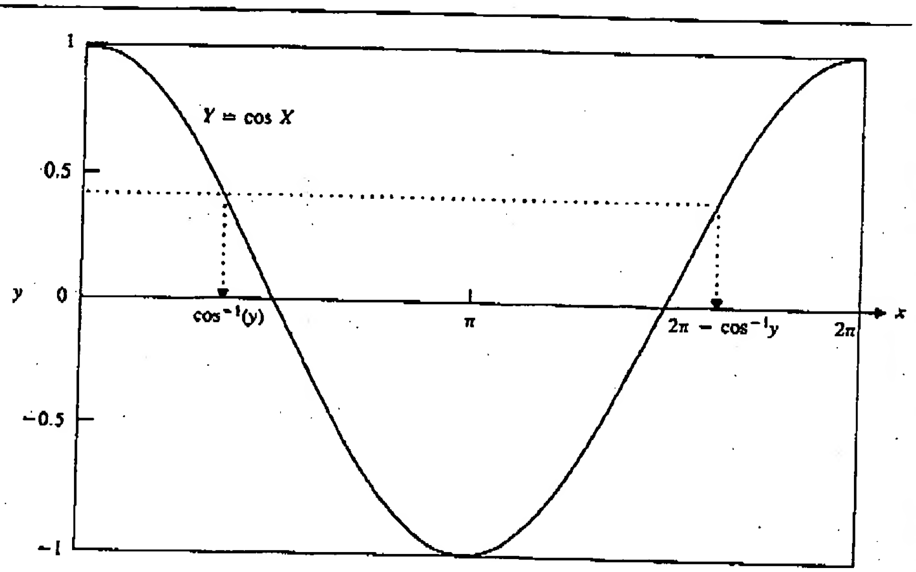
FIGURE 3.19
$y = \cos x$ has two roots in the interval $(0, 2\pi)$.



and since $f_X(x) = 1/2\pi$ in the interval of interest, Eq. (3.55) yields

$$f_Y(y) = \frac{1}{2\pi\sqrt{1-y^2}} + \frac{1}{2\pi\sqrt{1-y^2}}$$
$$= \frac{1}{\pi\sqrt{1-y^2}} \quad \text{for } -1 < y < 1.$$

The cdf of Y is found by integrating the above:

$$F_Y(y) = \begin{cases} 0 & y < -1 \\ \frac{1}{2} + \frac{\sin^{-1} y}{\pi} & -1 \le y \le 1 \\ 1 & y > 1. \end{cases}$$

Y is said to have the **arcsine distribution.**

3.6 THE EXPECTED VALUE OF RANDOM VARIABLES

In order to completely describe the behavior of a random variable, an entire function, namely the cdf or pdf, must be given. In some situations we are interested in a few parameters that summarize the information provided by these functions. For example, Fig. 3.20 shows the results of many repetitions of an experiment that produces two random variables. The random variable Y varies about the value 0, whereas the random variable X varies about the value 5. It is

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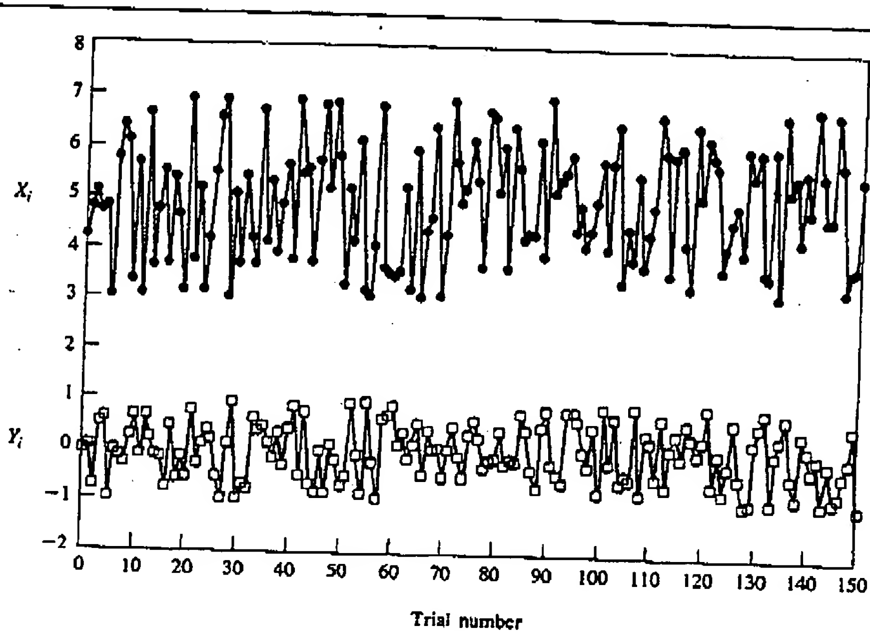

FIGURE 3.20
The graphs show 150 repetitions of the experiments yielding the random variable X and the random variable Y. It is clear that X takes on values centered about the value 5 while Y takes on values centered about 0. It is also clear that X is more spread out than Y.

also clear that X is more spread out than Y. In this section we introduce parameters that quantify these properties.

The Expected Value of X

The **expected value** or **mean** of a random variable X is defined by

$$E[X] = \int_{-\infty}^{+\infty} t f_X(t)\, dt. \tag{3.57}$$

If X is a discrete random variable, substitution of Eq. (3.21) into Eq. (3.57) yields

$$E[X] = \sum_k x_k p_X(x_k). \tag{3.58}$$

The expected value $E[X]$ is defined if the above integral or sum converges absolutely, that is,

$$E[|X|] = \int_{-\infty}^{+\infty} |t| f_X(t)\, dt \qquad < \infty$$

or

$$E[|X|] = \sum_k |x_k|\, p_X(x_k) \qquad < \infty.$$

There are random variables for which the above expressions do not converge. In

such cases, we say that the expected value does not exist. See Problems 71 and 72 for examples of such random variables.

If we view $f_X(x)$ as the distribution of mass on the real line, then $E[X]$ represents the center of mass of this distribution.

Equation (3.58) appeared in Chapter 1, Eq. (1.9), where it was pointed out that the arithmetic average of a large number of independent observations of a random variable X will tend to converge to $E[X]$. In this sense, the expected value of a random variable corresponds to our intuitive notion of the "average of X."

EXAMPLE 3.29
Mean of a Uniform Random Variable

The mean for a uniform random variable is given by

$$E[X] = (b - a)^{-1}\int_a^b t\,dt = \frac{a + b}{2},$$

which is exactly the midpoint of the interval $[a, b]$. The results shown in Fig. 3.20 were obtained by repeating experiments in which outcomes were random variables Y and X that had uniform cdf's in the intervals $[-1, 1]$ and $[3, 7]$, respectively. The respective expected values, 0 and 5, correspond to the values about which X and Y tend to vary.

The result in Example 3.29 could have been found immediately by noting that $E[X] = m$ when the pdf is symmetric about a point m. That is, if

$$f_X(m - x) = f_X(m + x) \qquad \text{for all } x,$$

then, assuming that the mean exists,

$$0 = \int_{-\infty}^{+\infty} (m - t)f_X(t)\,dt = m - \int_{-\infty}^{+\infty} tf_X(t)\,dt.$$

The first equality above follows from the symmetry of $f_X(t)$ about $t = m$ and the odd symmetry of $(m - t)$ about the same point. We then have that $E[X] = m$.

EXAMPLE 3.30
Mean of a Gaussian Random Variable

The pdf of a Gaussian random variable is symmetric about the point $x = m$. Therefore $E[X] = m$.

The following expressions are useful when X is a nonnegative random variable:

$$E[X] = \int_0^{\infty} (1 - F_X(t))\,dt \qquad \text{if } X \text{ continuous and nonnegative} \qquad (3.59)$$

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EXAMPLE 3.3
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Random Variable